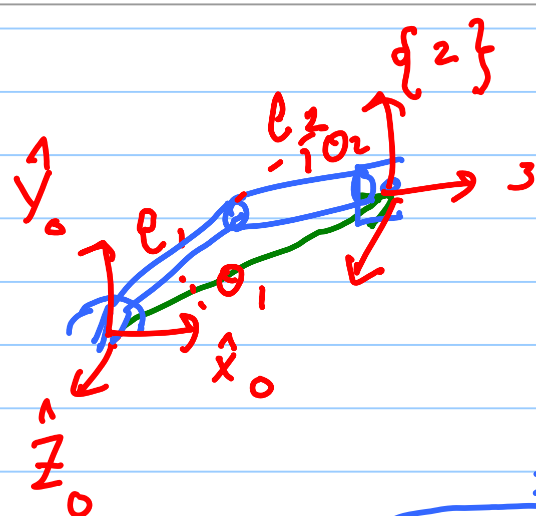
Lecture 6

Note Title 1/24/2012

$c_{12} = \cos(\theta_1+\theta_2)$

$s_{12} = \sin(\theta_1+\theta_2)$



$${}^0_2T = \left[\begin{array}{cc|c} c_{12} & s_{12} & l_1c_1+l_2c_{12} \\ -s_{12} & c_{12} & l_1s_1+l_2s_{12} \\ 0 & 0 & 0 \\ & & 1 \end{array}\right]$$

$\to {}^0P_{2ORG}$

Rot. around $\hat{z}$ $(\theta_1+\theta_2)$

$${}^0P_{2ORG} = \begin{pmatrix} l_1c_1+l_2c_{12} \\ l_1s_1+l_2s_{12} \\ 0 \end{pmatrix} \begin{matrix} x \\ y \\ \end{matrix}$$

$$\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix}$$

$${}^0\dot{P}_{2ORG} = v_2$$

$$\dot{x} = \frac{\partial x}{\partial \theta_1} \cdot \dot{\theta}_1 + \frac{\partial x}{\partial \theta_2} \cdot \dot{\theta}_2$$

$$\dot{y} = \frac{\partial y}{\partial \theta_1} \dot{\theta}_1 + \frac{\partial y}{\partial \theta_2} \dot{\theta}_2$$

Jacobian matrix

$$v_2 = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}$$

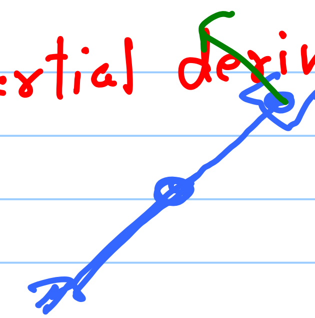
$$= \begin{pmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}$$

$$\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} \text{ Joint rate } \underset{J^{-1}}{\overset{J}{\rightleftarrows}} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$$

Cartesian vel

$$\mathbb{R}^n \longrightarrow \mathbb{R}^m$$

$n \times m$ matrix of partial derivatives



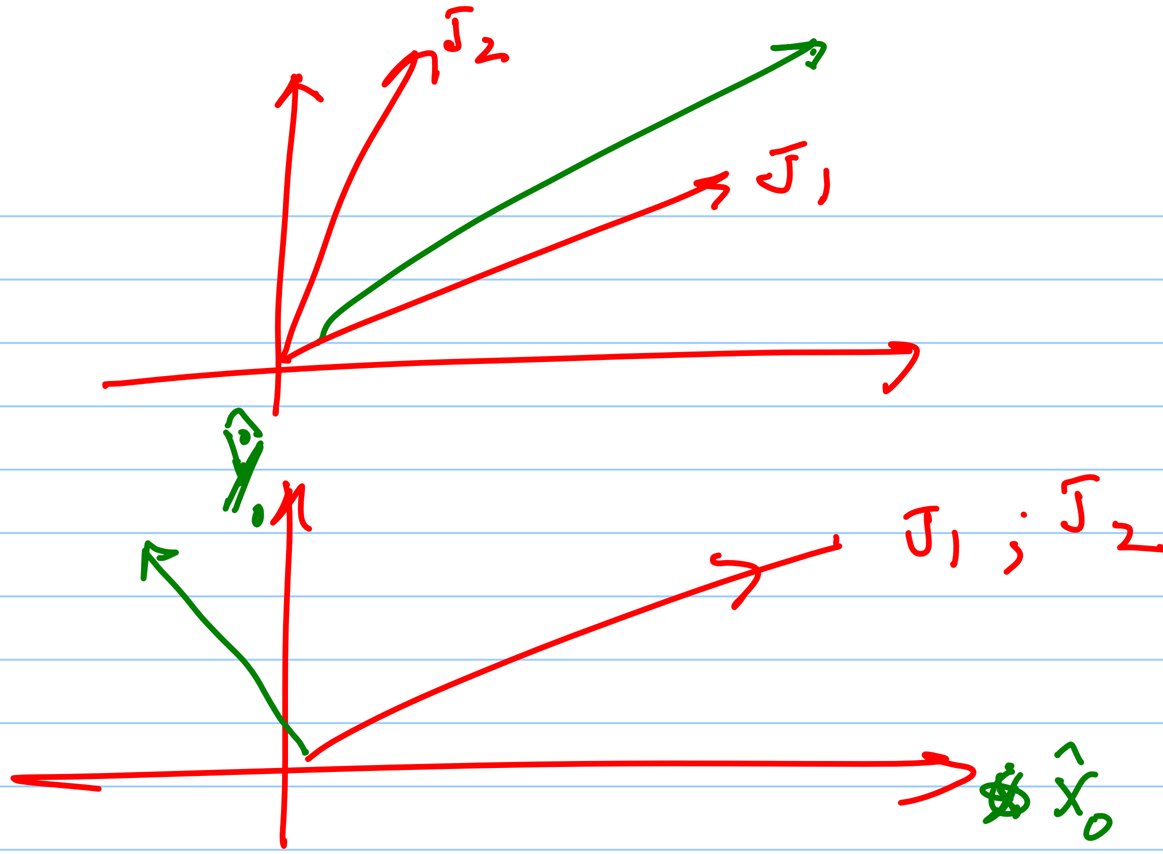
Inv. Comp. :

$\theta_2 = 0$

$$J = \begin{array}{cc} J_1 & J_2 \\ \multicolumn{2}{c}{\begin{pmatrix} -l_1 s_1 - l_2 s_1 & -l_2 s_1 \\ l_1 c_1 + l_2 c_1 & l_2 c_1 \end{pmatrix}} \end{array}$$

$$= \begin{pmatrix} -(l_1 + l_2) s_1 & -l_2 s_1 \\ (l_1 + l_2) c_1 & l_2 c_1 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = J_1 \dot{\theta}_1 + J_2 \dot{\theta}_2$$



J_2
J_1
$\hat{y}_0$
$J_1 ; J_2$
$\hat{x}_0$

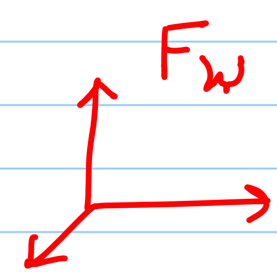
1) $\boxed{\text{Configuration space}}$

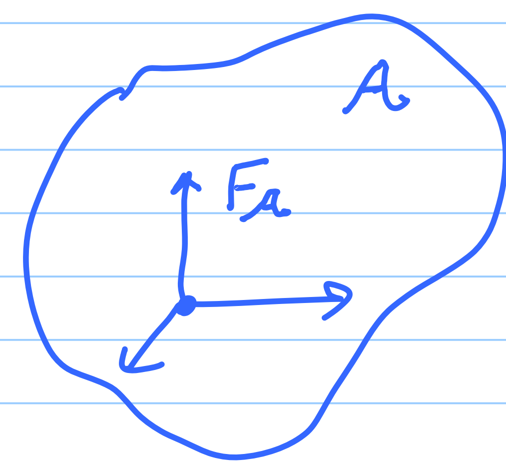
2) metrics C-space

3) Basic path planning

Rigid body :

Global frame





geom of A is known

"Configuration" of $\mathcal{A}$ is defined as the min. parameters needed to specify the exact position of all points $a \in \mathcal{A}$ in the Global frame.

Having fixed F_A on A, this boils down to the min. # of parameters needed to specify pos. + orient. of $\cancel{A}$ F_A w.r.t. F_W

[three parameters for pos., 3x3 Rot. matrix for orient]

q configuration of the rigid body

Configuration space: C space space of all q's.

$$SE(3) = \left(\begin{array}{c|c} R_{3\times 3} & P_{3\times 1} \\ \hline 0 & 1 \end{array}\right)$$

$$q = \begin{pmatrix} x \\ y \\ z \\ \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$\boxed{\mathbb{R}^3 \times SO(3)}$

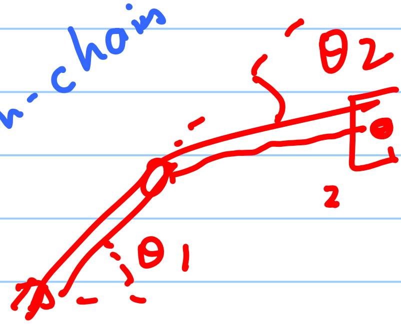
Locally

locally $\mathbb{R}^3$

$$\mathbb{R}^N \times SO(N)$$

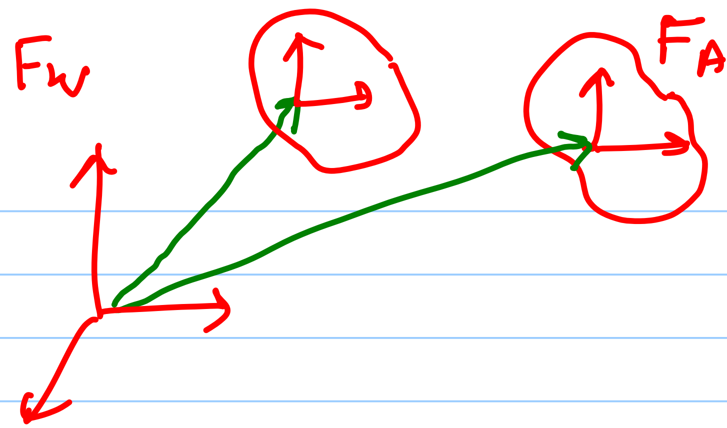
$N = 2$ $\quad \mathbb{R}^2 \times SO(2) \quad \mathbb{R}^2 \times S^1$

Open-chain



$\underbrace{S^1 \times S^1}$ $\sim$ <u>Torus</u>

rigid body that can only translate in plane.



holonomic

$$g_i(q) = 0$$

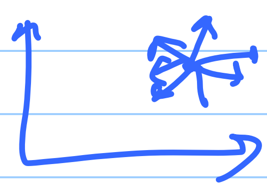
Constraints :

|| in config. space

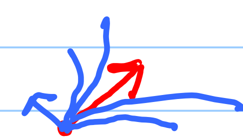
non-holonomic

$$g_i(q, \underline{\underline{\dot{q}}}) = 0$$

non-integrable



Car like robots



closed chain :

Grubler's formula

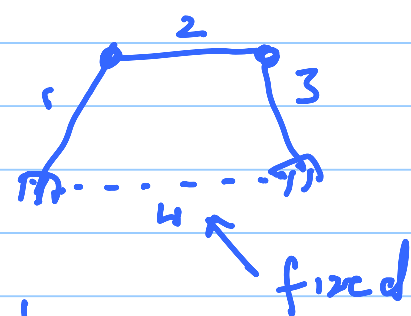
$M =$ min # of param. needed

"mobility" (dof)

or

# of deg of freedom

dim of C-space



f_i = # dofs for each joint

k links

n joints

$N - f_i$

Constraints at each joint

N = # of dofs for each link.

$$M = N(k-1) - \sum_{i=1}^{n} (N - f_i)$$

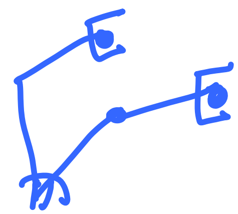
$k = 4, \quad N = 3, \quad f_i = 1 \qquad n = 4$

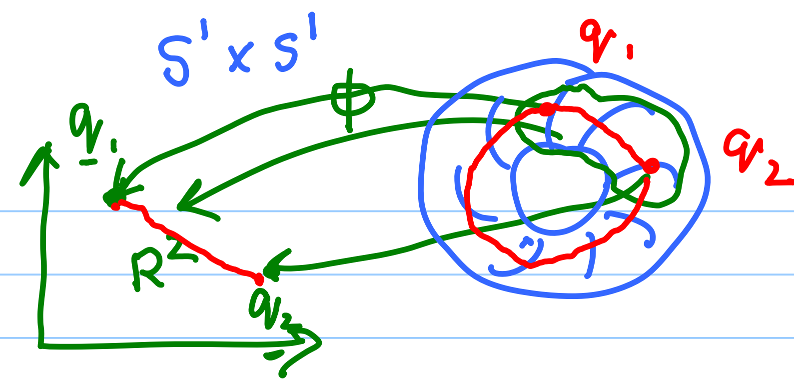
$$\Rightarrow M = 3(4-1) - \sum_{i=1}^{4} (3-1)$$

$$= 1$$

2) metrics in C-space?

|| "how far apart are two diff. configs"?





① C-space is a m-dim. manif.

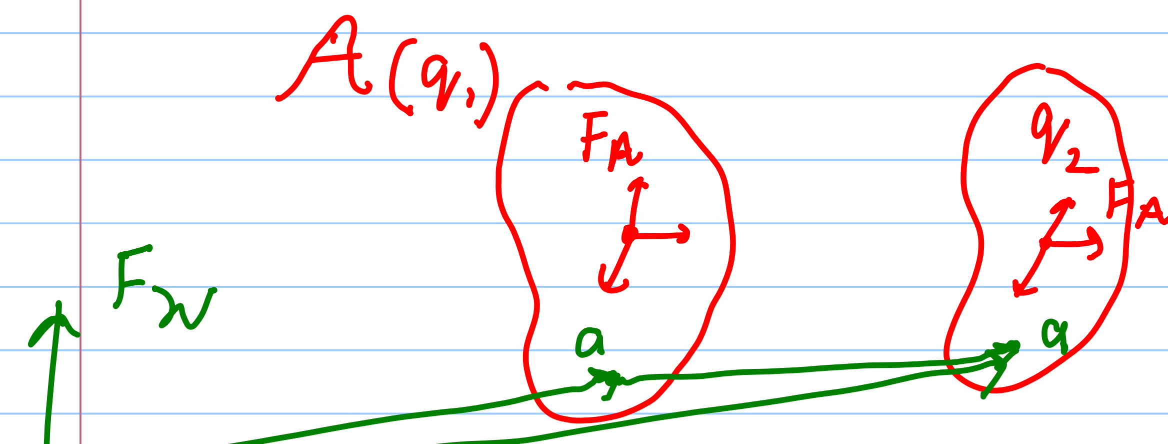
we could use the "parameter space"

Euclidean distance. $\underline{q}_1 = \begin{pmatrix} \theta_{11} \\ \theta_{12} \end{pmatrix}$

$d(q_1, q_2)$ $\qquad \underline{q}_2 = \begin{pmatrix} \theta_{21} \\ \theta_{22} \end{pmatrix}$

$= \sqrt{(\theta_{11} - \theta_{21})^2 + (\theta_{12} - \theta_{22})^2}$

2) "max dist. travelled by any point on the ~~bo~~ robot in going from q_1 to q_2.



$$d(q_1, q_2) = \max_{a \in \mathcal{A}} \| a(q_1) - a(q_2) \|$$

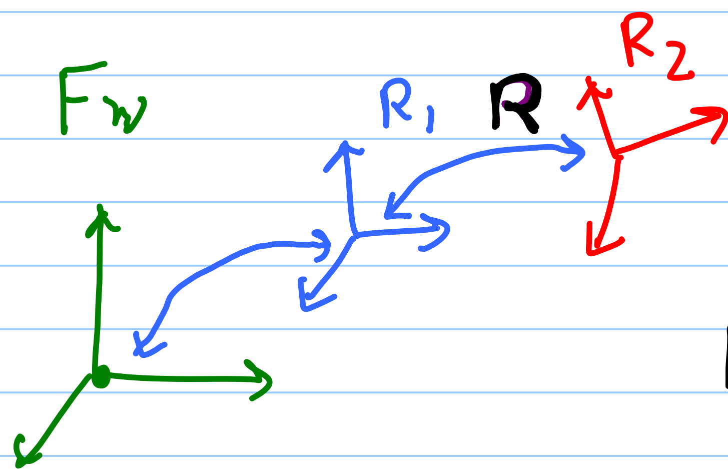
3) <u>Rigid body:</u> $\mathbb{R}^3 \times SO(3)$

$$q_1 = (P_1, R_1) \longrightarrow \begin{pmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{pmatrix}$$

$$q_2 = (P_2, R_2) \longrightarrow \begin{pmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{pmatrix}$$

$$d(q_1, q_2) = d(P_1, P_2) + \lambda \left\| \left(\begin{pmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{pmatrix}, \begin{pmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{pmatrix}\right.\right.$$

$$d(R_1, R_2) \overset{?}{=}$$



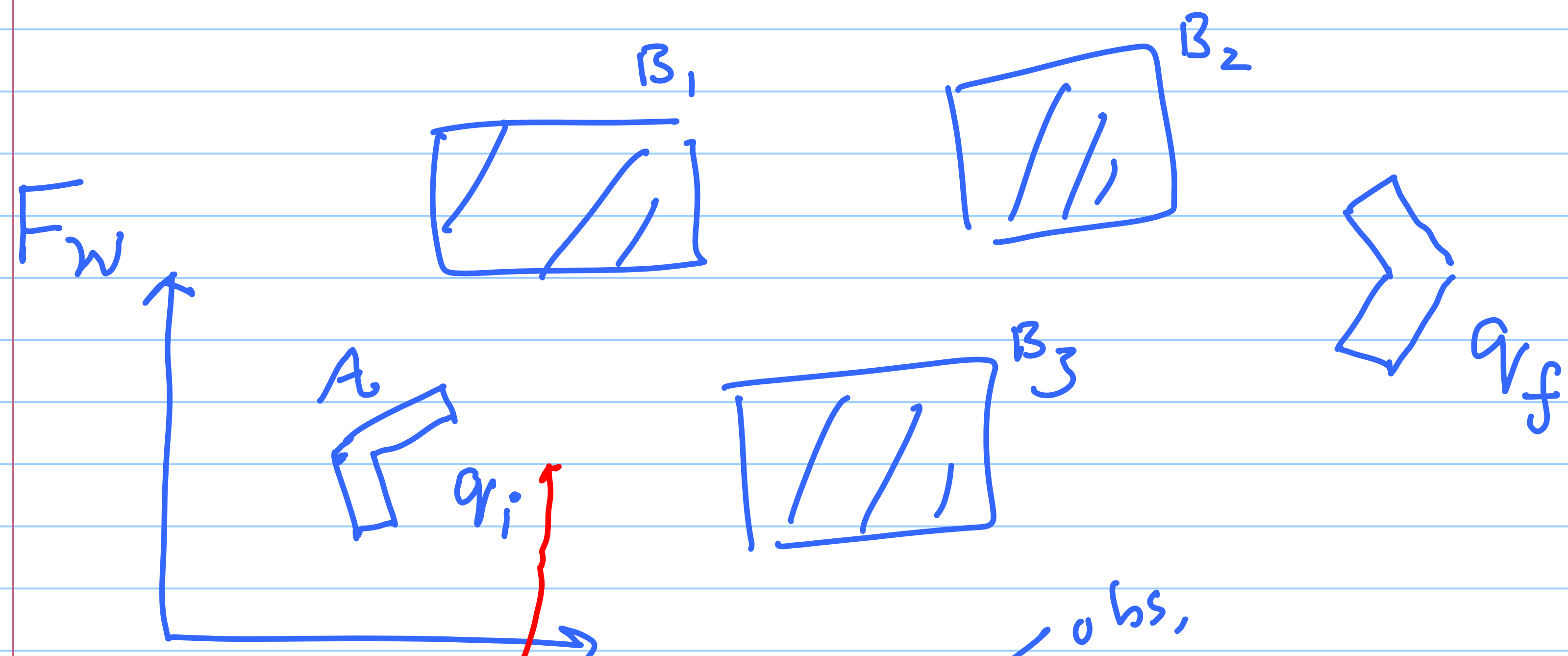
$$R R_1 = R_2$$

$$\Rightarrow \underline{\underline{R}} = R_2 R_1^{-1}$$

$$\begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix} \theta$$

$$\boxed{d(R_1, R_2) = \theta}$$

BASIC Path Planning Problem



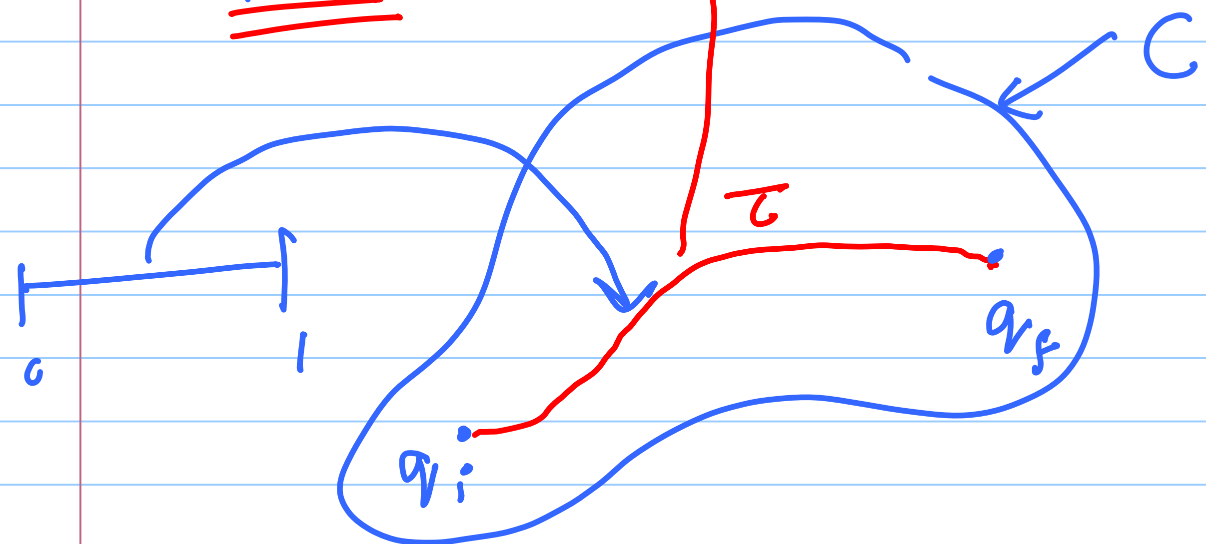
obs,

<u>Given</u>: B_i's, A, q_i, q_f

Problem: determine a collision-free

τ

path that connect q_i to q_f.



$A(q) \cap B_i = \phi$

$q \in \tau$